647510 **BSc3MatC0301** **Seat No :_________**

B.Sc. Semester - 3 (*CBCS*) Examination
Oct/Nov. -2018
MATHEMATICS(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

1(A)Answer any one: **(07)**

(1)If $\lim_{n\to\infty} a_n = l$ then prove that $\lim_{n\to\infty}\left(\frac{a_1+a_2+a_3+\cdots a_n}{n}\right) = l.$

(2)Prove that: $\lim_{n\to\infty} \sqrt[n]{a} = 1\,; a > 0.$

(B)Answer any one: **(04)**

(1)Prove that the sequence $\{S_n\}$ defined by $S_1 = \sqrt{7}, S_{n+1} = \sqrt{7+S_n}$ converges to the positive root of the equation $x^2 - x - 7 = 0.$

(2) Show that the sequence$\{S_n\}$ is divergent, where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

(C)Answer any one: **(03)**

(1) Define: Convergence of sequence and discuss the convergence of sequence$\{\sqrt{n+1} - \sqrt{n}\}$

(2)Define: Cauchy sequence and state the Cauchy's general principle of convergence.

2 (A) Answer any one: **(07)**

(1) State:Raabe's test and using Raabe's test, discuss the convergence of

$$1 + \frac{3}{7}x + \frac{3\cdot 6}{7\cdot 10}x^2 + \frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3 + \cdots$$

(2) State: D'Alembert's ratio test and using D'Alembert's ratio test, discuss the convergence of

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots$$

(B) Answer any one: **(04)**

(1) Show that the series $\sum(-1)^n\left(\sqrt{n^2+1} - n\right)$ is conditionally convergent.

(2) Using comparison test show that the series $\sum\left(\sqrt[3]{n+1} - \sqrt[3]{n}\right)$ is divergent.

(C) Answer any one: **(03)**

(1) Define: Geometric series and test the convergence of the series

$$1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \cdots$$

(2) State the test for convergence of power series.

3 (A) Answer any one: **(07)**

(1)If $\bar{f}$ and $\bar{g}$ are vector functions defined on the domain D whose first order derivatives exist then prove that:

$$div(\bar{f} \times \bar{g}) = \bar{g}\cdot curl\bar{f} - \bar{f}\cdot curl\bar{g}$$

(2) If $\bar{f}$ and $\bar{g}$ are irrotational functions on D then show that $\bar{f} \times \bar{g}$ is a solenoidal function

(B) Answer any one: **(04)**

(1) In usual notation prove that: $\nabla^2(log r) = \frac{1}{r^2}$, where $r = |\bar{r}| = \sqrt{x^2+y^2+z^2}$

(2) Prove in usual notations, $div(r^n\bar{r}) = (n+3)r^n$

(C) Answer any one: **(03)**

(1) If $\emptyset(x,y,z) = x^3y + xy^3 + xyz$ then find $\nabla\emptyset$.

(2) If $\bar{f} = (x^3, y^3, z^3)$ then find $curl\bar{f}$.

4 (A) Answer any one: **(07)**

(1) Prove that : $\iint\limits_{D} \sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}} = \frac{\pi}{2}(\pi-2)ab$, where D is ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

(2) Evaluate: $\iiint\limits_{R} (x^2+y^2+z^2)^5 dxdydz = \frac{4\pi a^{13}}{13}$, where $x^2+y^2+z^2 = a^2$

(B) Answer any one: **(04)**

(1)Evaluate: $\iint xydxdy$ over the region in the positive quadrant for which $x + y \le 1$.

(2) Evaluate: $\iint\limits_{R} [xy(1-x-y)]^{\frac{1}{2}} dxdy$, taken over the area of the triangle with sides $x = 0, y = 0, x + y = 1$.

(C) Answer any one: **(03)**

(1) Evaluate: $\int_0^9 \int_{\sqrt{y}}^3 sin(\pi x^3)dxdy$, by changing the order of integration.

(2) Define: Triple integration.

5 (A) Answer any one: **(07)**

(1) State and prove Green's theorem for a plane.

(2) In usual notation prove that: $\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}, p > 0, q > 0.$

(B) Answer any one: **(04)**

(1) Find $\oint\limits_{C} \overline{u_t}\ ds$, where $\overline{u_t} = y^2\hat{\imath} + xy\hat{\jmath}$ and C is a square with vertices $(1,1), (-1,1), (-1,-1)\ and\ (1,-1)$.

(2) Find: $\iint\limits_{S} x^2dydz + y^2dzdx + 2z(xy-x-y)dxdy$, where $S: 0 \le x, y, z \le 1$ a solid surface.

(C) Answer any one: **(03)**

(1) State: Stoke's theorem and find $\int_c\ 2xy^2zdx + 2x^2yz + (x^2y^2-2z)dz$ Where c is $x^2 + y^2 + z^2 = a^2$.

(2) In usual notation, show that: $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \sqrt{2}\pi.$

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